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TOWARDS A QUANTUM PUMP OF ELECTRIC CHARGES AND CURRENTS

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Towards a quantum pump of electric charges and currents

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ABSTRACT: Here we give a critical examination of the possibility of realizing a quantum pump of electric charges and currents. The physics is based on the theory of quantum adiabatic particle transport initially due to Thouless. We present theoretical guide lines on the experimental conditions for observing this phenomenon. We argue that potentially very accurate quantization of charge transport or electron current can be achieved, once these conditions are approximately satisfied. An example of experimental set-up is outlined to demonstrate the practical possibilities. Some comments are also made on the scientific significance of such a quantum device.

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We have a quantum voltage standard from the Josephson effect, and we also have a quantum resistance standard from the quantum Hall effect. As reported by Taylor in Physics Today, these quantum standards will be put into practical use at the beginning of 1990 by international agreements.[1] Can we also have a quantum standard for electric charges or currents? Here we propose a possible quantum charge pump that has precise control on the amount of pumped charge or electric current. As will be explained below, this idea may be realized through the effect of quantized adiabatic particle transport initially due to Thouless.[2]

Ideally, we consider a one dimensional noninteracting electron gas in a potential. $U(x,t)$, with spatial and temporal periodicities of a and τ, respectively. We assume zero temperature, adiabatic time variation of the potential, and an integer number of filled Bloch bands of the instantaneous Hamiltonian,

$$H(t) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x,t). \tag{1}$$

Then, according to Thouless, the number of electrons transported through any cross section during time τ is an integer, provided that the Fermi gap[3] remains open at all times. The corresponding DC electric current is then quantized as

$$I = ne\nu, \tag{2}$$

where $\nu = 1/\tau$ is the operating frequency of the potential.

We will first give a brief exposition of the physics behind this effect, and answer some important questions in the above model. We will then present more realistic considerations, and outline a possible experimental set-up. Finally, we will discuss the scientific significance of such a device.

The quantization of particle transport can be easily understood in the simplest case of a rigidly sliding potential $U(x - vt)$, where the temporal period is related to the spatial

period as $\tau = a/v$. The adiabatic theorem and the Galilean principle ensure that the electron number density of the filled bands shifts all together with the potential. The integral of the number density over the spatial period l is an integer n for n filled bands,[4] which is also the number of electrons transported in time τ.

In the more general case where the Galilean principle does not hold, the result is not trivial at all. The original proof of quantization was based on topological arguments regarding the phase of a wave function in multiply connected parameter space. Here we present a new and physically more transparent proof.[5] Consider, for simplicity, the case that only the lowest band is filled. The corresponding many body state is a determinant of Bloch waves, but it can also be written as a determinant of single particle Wannier states.[6] The particle density can then be written as

$$\rho(x,t) = \sum_{-\infty<j<\infty} |W(x-ja,t)|^2, \tag{3}$$

where $W(x,t)$ is a Wannier function for the filled band of the instantaneous Hamiltonian $H(t)$. It is convenient to define $W(x,t)$ as an eigenstate of the position operator projected on to the subspace of the Bloch band,

$$\hat{P}(t)\hat{x}\hat{P}(t)\,W(x,t) = l(t)\,W(x,t), \tag{4}$$

where $\hat{P}(t)$ is the projection operator. The eigenposition, $l(t)$, is also the center of mass of the density profile $|W(x,t)|^2$.[7] Such a Wannier function is exponentially localized in space. The other eigenstates of $\hat{P}(t)\hat{x}\hat{P}(t)$ are of the form $W(x-ja,t)$, with their eigenpositions given by $l_j(t) = l(t) + ja$. It can be shown that, if the Fermi gap does not close, $l(t)$ varies continuously with $U(x,t)$.[5] We can therefore follow the positions of the Wannier functions. Since the set, $\{l_j(t)\}$, of eigenpositions must return to its initial configuration when $U(x,t)$ returns its initial form, each Wannier state must have shifted its center of mass by $l(t+\tau) - l(t) = na$. The particle transport, namely the time integral of the current due to the motion of these localized densities, is just equal to n, thus quantized.

In an article of the author.[8] the integer of particle transport was calculated for a number of non-trivial cases. One important question asked there was the following: suppose the potential $U(x,t)$ consists of an array of mutually disjoint local potentials, can bounded movements of the local potentials induce global transport of the particles? In other words, can the particle transport be a nonzero integer? One type of local movements considered was coherent positional oscillations as in a traveling sound wave. The answer is yes in general. The other type was phase-coherent amplitude oscillations with a travelling envelop. The answer is also positive in the generic case.

Here we examine an example of the latter type, partly to illustrate ideas, and partly for its relevance to later discussions. Suppose the total potential is given by

$$U(x,t) = \sum_{-\infty<j<\infty} A_j(t)\, u(x - ja/3), \tag{5}$$

where $u(x)$ is a local potential (a square barrier, say), with the amplitudes varying coherently in time as

$$A_j(t) = C + C_1 \cos[2\pi(j/3 - t/\tau)]. \tag{6}$$

The spatial period of $U(x,t)$ is a and contains 3 local potentials. Fig.1 illustrates how the potential progresses in time. It is seen that, after a third of the temporal period $\tau/3$, the potential configuration has been shifted to the right by $a/3$. Each of the Wannier functions in a given Bloch band must have shifted a distance of $ma + a/3$, where m is an integer. The total shift during a whole period τ is $3ma + a$. The corresponding particle transport is then $3m + 1$, a nonzero integer. In fact, if the Fermi gap is mainly determined by the corresponding Fourier component of the potential, then the integer of particle transport is 1, -1 or 0, for the lowest one, two or three filled bands, respectively.[8] It is easy to understand, by following the positions of the minima of the potential shown in Fig.1, how the particle density in the lowest band is squeezed to the right.

Having briefly described the physics of quantized charge transport in the ideal situation,

we now proceed to consider a number of major issues that one must face in reality. First of all, the formula (2) of the quantized current involves an operational frequency ν. However, as for the voltage standard using AC Josephson effect, the accuracy in the frequency control can be very high. Thus, the main question is the correction to the integer. From experience in the theoretical development for the integral quantum Hall effect, we expect that a finite Fermi gap will make the electrons inert to all kinds of perturbations: thermal activation, nonadiabatic excitation, static disorder, many-body interaction, finite size, and etc. The correction due to thermal activation will be of the order of $e^{-\Delta/(KT)}$, which can be very small if the thermal energy gets smaller than the Fermi gap Δ. The correction due to nonadiabatic excitations is an exponential of $-\Delta/(h\nu)$,[9] and can therefore be easily controlled. It has also been shown by Niu and Thouless[10] that, quantization of charge transport is not affected by disorder and many body interaction, so long as the adiabatic ground state of the system is separated from the excitations by a finite energy gap. Finally, current leads at the ends of the system will certainly close the Fermi gap, but the states in the gap are localized in the leads. In order for these states to have an effect, they must tunnel all the way from one end to the other of the system. The correction to quantization is therefore of the order of $e^{-L/l}$, where L is the system size, and l is the localization length of the edge states. In summary, once the system lies near the perfect situation, every conceivable corrections to the quantization is either identically zero, or exponentially small in the perturbation. Thus, the effect of quantized charge transport can in principle lead to a potentially very accurate standard for electric charge and current measurements.

The analogy with the quantum Hall effect goes further. It was shown by Laughlin[11] that the integer of the quantized Hall conductance is equal to the number of electrons transported from one edge of a cylinder to the other when a magnetic flux through the center of the cylinder is adiabatically increased by a flux quantum. The number of trans-

ported electrons is an integer, because, in the bulk, the extended states below the Fermi energy map to one another, and the localized states about the Fermi energy map to themselves. Consider in our case a situation that the Fermi energy lies in a range of localized states, which map to themselves when the potential evolves adiabatically back to its initial configuration. We will call such states as impurity-locked states, and their energy range the impurity-locked mobility gap.[12] The particle transport should still be quantized by the Laughlin's argument. Since the impurity-locked states contribute nothing to the particle transport, thermal or non-adiabatic smearing of the Fermi-energy within the impurity-locked mobility gap should introduce no correction to the integer of particle transport. Corrections due to excitations beyond the impurity-locked mobility gap Δ_i should be exponentially small in $\Delta_i/(KT)$ or $\Delta_i/(h\nu)$. Errors due to Mott-type inelastic hops over the localized states are also exponentially small.

The lesson of the quantum Hall effect also tells us that the one-dimensionality of the system is not necessary. We may consider a quantum wire with several lateral modes below the Fermi energy, so long as we have a big enough Fermi gap to beat down all types perturbations.

How can one create a potential that can induce a nonzero and quantized particle transport? Here we suggest an example to illustrate the practical possibility. The details will be presented elsewhere.[13] The set-up is illustrated in Fig.2. We first make a quantum wire on a substrate. We then connect to the wire two sets of voltage leads located at $\{x = na\}$ and $\{x = na + a/3\}$, respectively, where x is the coordinate along the wire, and a would be the spatial period. The leads within a set should be identical. Let the first set lie on the left of the wire, with the individual leads connected together internally, and with just one common lead L_1 coming out of the device. The second set will be put on the right of the wire, connected internally and coming out the device with a common lead L_2. We then ground the substrate, and apply to L_1 and L_2 the voltages $V_1(t)$ and $V_2(t)$,

respectively. The whole quantum wire is also going to be biased by an additional potential $V_b(t)$ with respect to the substrate. We choose these voltages to be such that

$$V_j(t) = A(t) + B\cos[2\pi(j/3 - t/\tau)], \quad j = 1, 2, 3 \tag{7}$$

where $V_3(t)$ is the voltage at the positions $\{x = na + 2a/3\}$ (marked gray in Fig.2) along the quantum wire. The common term $A(t)$ should be chosen to regulate the zeropoint of the electronic energy spectrum in the quantum wire, such that a given energy gap always contains in its middle the Fermi energy (or the chemical potential maintained by the substrate).

The potential energy created along the quantum wire is then similar to the one given in (5) and (6). The actual potential does not need to be perfect, as has been emphasized before. It is however important to always lock the Fermi energy in a gap. How? Suppose we want to find the appropriate value of $A(t)$ at $t = t_0$. Take a static voltage configuration:

$$V_j = A' + B\cos[2\pi(j/3 - t_0/\tau)], \quad j = 1, 2, 3. \tag{8}$$

Measure the conductance of the wire at a low enough temperature. By looking at the pattern that the conductance changes with A', we should know for which value of A' the Fermi energy is in the desired gap. We then assign the appropriate value of A' to $A(t_0)$.

Suppose the wire has only one lateral mode below the Fermi energy. The energy gap above the first Bloch band should be of the order of $\Delta = |eB|$, if the thickness of each of the leads is about $a/6$. The condition for $\Delta > K_B T$ can be fulfilled for $T < 10K$ if $|B|$ is greater than a millivolt. For a frequency as high as $\nu < 10^{12}$ Hertz, the adiabatic condition, $h\nu << \Delta$, is well satisfied.

We may take the period a of the order of $1000\AA$, and we need about 20 or so such periods with good periodicity. More periods and better periodicity are preferred. Finally, to observe a nanoampere current, we need $\nu \sim 10^{10}$ hertz, a frequency corresponding to a

wave length of 3cm of an electromagnetic wave. Lower frequency can be used to produce the same amount of current, if more quantum channels (additional lateral or longitudinal modes, or more quantum wires) are used.

If the requirement of high frequency cannot be satisfied at the present stage of technology, we can nevertheless stick to a lower frequency. We may not observe a steady current, but we can store the pumped charges to a capacitor in series with the quantum wire. We then discharge the capacitor at time intervals τ_i bigger than τ, and measure the current pulses. The relaxation time τ_c of the capacitor should be smaller than τ_i. The current at the pulses is enhanced by a factor of τ_i/τ_c. The integrated charge of a pulse should be quantized in units of $Q = e\tau_i/\tau$.

From the above considerations, the observation of the quantized charge transport should not be too difficult, and is certainly possible. Better designs from experienced experimentalists are expected, once the possibility has been demonstrated. It is emphasized again that great potential of refining the accuracy of measurements exists due primarily to the exponential dependence of errors on small perturbations.

How can we make use of such a quantum device? Here we just list a few foreseeable applications. First of all, the device is a high resistance low-current source. The resistance is exponentially large in the length of the quantum wire, until the voltage becomes comparable to Δ/e. Secondly, it can serve as a quantum pump of electric charges. The amount of pumped charges can be conveniently and accurately controlled by the number of operation cycles. Consequently, we can use it to measure capacitances with high precision.[14] Thirdly, the device can be used as direct standards for measurement on electric charges and currents, assuming the electron charge is accurately known. Alternatively, it can be used for the measurement of the fundamental electron charge e.[15] Finally, the device can, of course, be used to study physics.

If the device as a current standard is realized, then we have completed the triangle of voltage, resistance, and current involved in the Ohm's Law, with three direct quantum standards derived from three physically different quantum effects: the Josephson effect, the quantum Hall effect, and the effect of quantum adiabatic particle transport. The three quantum standards are not totally independent. They are in fact related by a 'quantum Ohm's Law': we can combine any two of them to mimic the third.[16]

On a more fundamental level, the quantum device generates the charge quantum e, a Josephson device generates the magnetic flux quantum $h/2e$, whiie a quantum Hall system generates the resistance quantum h/e^2. These satisfy another Ohm's Law: $e \cdot (h/e^2) = 2 \cdot h/(2e)$. Therefore, these devices provide three different ways to measure two independent fundamental constants. We can check whether the fundamental constants really appear the same in different systems, although they should be so according to the present theoretical understanding of these systems.

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2. D.J.Thouless, *Phys. Rev. B* **27**, 6083, (1983); A.Zee, *Phys. Lett.* **135B**, 307, (1984).

3. Here and hereafter, Fermi gap refers to the energy gap between the filled and empty bands of H(t).

4. Here and hereafter, a factor of 2 due to the spin degeneracy is omitted, for the sake of concentrating on the essentials.

5. Details will be given elsewhere.

6. The Wannier functions are essentially the Fourier transforms of the Bloch waves with respect to the wave number k. They can be made exponentially localized by suitable choice of the phase of the Bloch functions. See, for example, W.Kohn, *Phys. Rev.* **115**, 809, (1959).

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11. R.B.Laughlin, *Phys. Rev. B* **23**, 5632, (1981).

12. In the analogy, the extended states in the Hall problem corresponds to here the

states that are not locked by the impurities, although all the states may be localized at a given time.

13. Q.Niu and K.Ensslin, in preparation.

14. Connect a capacitor in series with the quantum charge pump. Operate the charge pump for a time t at a frequency ν. The total charge on the capacitor is then $ne\nu t$, where n is an integer. Measure the voltage, V, across the capacitor. The capacitance is then given by $ne\nu t/V$.

15. Pump a given number of electrons, then measure their total charge by other methods.

16. Thouless, private conversation.

FIGURE CAPTIONS

(1) The time evolution of the potential $U(x,t)$ in Eq.(5) during the the first third temporal period.

(2) The quantum wire and the voltage leads.

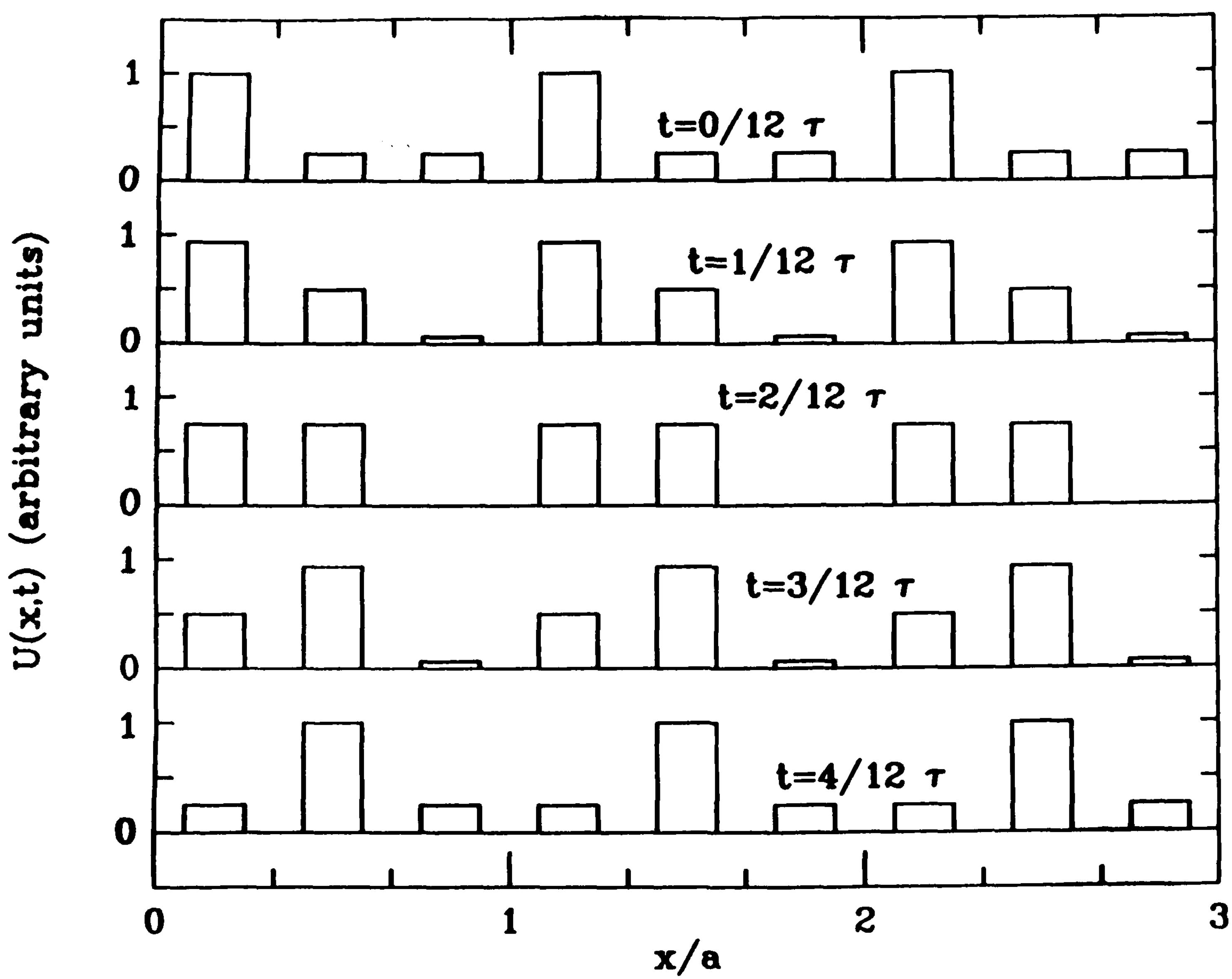

Fig. 1

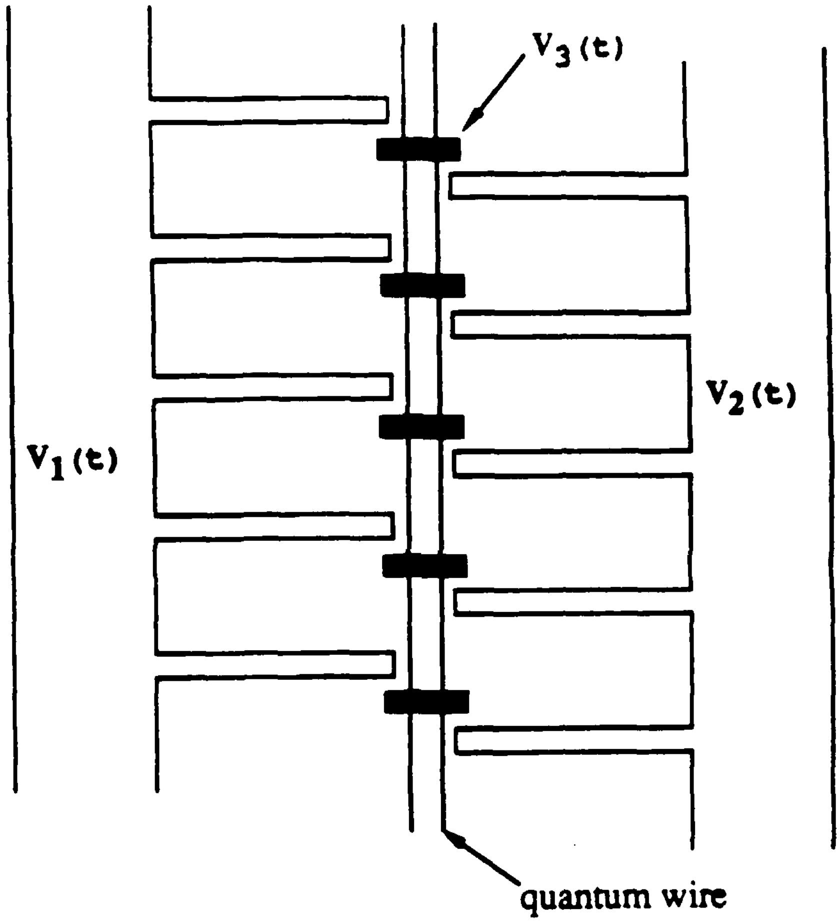

Fig. 2